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DYNAMICAL PROPERTIES OF STELLAR CORONAS AND STELLAR WINDS III. THE DYNAMICS OF CORONAL STREAMERS*

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Abstract

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Observations suggest that the solar corona may perhaps consist of thin radial filaments and streamers, rather than being a homogeneous atmosphere. The filaments and streamers presumably are confined by the magnetic field, along whose lines of force they lie. To investigate the consequences of a filamentary corona for the solar wind the dynamics of an idealized corona composed of gaseous streamers, with vacuum between the streamers, is worked out neglecting solar rotation etc. The principal conclusions are that the streamers occupy most of space at large radial distance from the sun, no matter how small a fraction they may occupy at the sun; the velocity of the solar wind is not greatly affected for a given coronal temperature distribution; the mean coronal density is not affected by more than a factor of two. Altogether, then, except for the presence of some thin interstreamer regions, there is very little effect on the solar wind at large distance from the sun.

The possibility of instability of coronal streamers is looked into and it is shown that, if the corona is composed principally of streamers, the Helmholtz instability may be expected to disorder and mix up the streamer and interstreamer regions beyond some distance O (1 a.u.) from the sun, which might perhaps be the origin of some of the disorder observed by Mariner.

More quantitative observational information of the solar corona and/or of the structure of the solar wind in toward the sun will have to be available before more quantitative statements can be made concerning the role of streamers in the solar wind.

I. INTRODUCTION

There is a great deal of evidence that the solar corona is composed of a large number of fine streamers or filaments. This paper explores the effect of a filamentary corona on dynamics of the solar wind. *To illustrate what may be expected, the extreme* case is considered wherein the streamers contain all the coronal material, with no gas between. The actual situation lies somewhere between this extreme and the opposite extreme of a continuous distribution of gas. Observations do not yet permit construction of a quantitative model of coronal streamers. The calculations given here show that, as one would expect, the streamer corona has the same fundamental property of expansion to supersonic velocity that was shown earlier for the smoothed corona (Parker, 1958b, 1960). *Given a mean coronal density and given a coronal temperature profile* $T(r)$, the <u>mean</u> solar wind velocity and density do not seem to be affected greatly. The principal effect of fine streamers in the corona appears to be the interesting possibility of the production of irregularities throughout the plasma and magnetic field in interplanetary space.

It is evident from observations of the solar corona during eclipses of the sun that the corona is composed of filaments and streamers that extend far into space. The degree to which the density and temperature of the coronal atmosphere is concentrated into filaments has not yet been determined with precision. The indications are (see discussion and references in van de Hulst, 1953 and Vsekhsvjatsky, 1963) that the temperature in the denser filaments is little, if any, higher than outside, whereas the density may perhaps be as much as ten times higher. The only known means for confining the observed streamers is the solar magnetic field. Hence it is generally assumed that the filaments, or streamers, lie along the lines of force of the extended solar magnetic field.

There is some difference of opinion as to the dynamical properties of the streamers and their role in solar corpuscular emission. Several workers in the field, such as Mustel and Vsekhsvatsky, have held the view for some time that the streamers play a key role in the structure of the corona and in the emission of corpuscular radiation into space. The reader is referred to the literature for drawings and photographs of coronal streamers (Astronomer Royal, 1927; van Biesbroeck, 1953; Kiepenheuer, 1953; Mustel, 1963; Vsekhsvjatsky, 1963) and for discussion of some ideas concerning their dynamical properties (van de Hulst, 1953; Mustel, 1963; Vsekhsvjatsky, 1963; Mogilevsky, 1963; de Jager, 1963, and references therein).

The present study considers the dynamical properties of the streamers, which extend more or less radially outward into space.* The overall hydrodynamic expansion of a smoothed corona has been considered elsewhere, where it was shown that the corona carries with it into space** the magnetic lines of force of the general solar magnetic field (Parker, 1958b, 1963a). The observed radial nature of the general solar magnetic field and of the streamers near the sun is a direct consequence of this. The visible streamers represent the outward extension

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*Re-entrant filaments are observed in some portions of the corona, of course, particularly in association with active regions. The discussion here does not deal with such closed systems, but considers only the open ended filaments, which are approximately radial.

** The presently observed solar wind strength suggests that the solar wind, and hence the magnetic lines of force, extend to some distance in excess of 10 a.u., where the resistance of the interstellar gas and magnetic field is met (Parker, 1961).

into space (along the lines of force) of density and/or temperature variations existing at the feet of the magnetic lines of force in the low corona and chromosphere. Some interesting and plausible ideas concerning the origin of the density and temperature variations in the low corona and chromosphere are expressed in the references cited above. We offer no new opinion here as to the origin of the variations, but regard the variations as a given boundary condition at the feet of the magnetic lines of force. The high electrical conductivity of the coronal gas constrains the gas to motion along the lines of force, thereby projecting the density variations outward into space. The close confinement of both the ions and electrons by the magnetic field provides thermal insulation between neighboring filaments, thereby projecting the temperature variations outward into space. The close confinement of the ions and electrons has the well known effect of greatly reducing the effective lateral viscosity (see discussion in Parker, 1962) between neighboring regions. Thus to a first rough approximation the individual streamers are regarded as separate hydrodynamic tubes of flow, with their character determined largely by the conditions in the low corona. The principal interaction between neighboring filaments is the lateral hydrostatic pressure, which must of course be in balance for stationary conditions.

Consider then the stationary expansion of a filament lying along the general magnetic field $\mathbf{B}$. Neglect the rotation of the sun. Let l denote distance measured along a given flow line and $\Phi(l)$ the gravitational potential per unit mass. For stationary conditions the flow velicity $\mathbf{v}$ is parallel to the magnetic field $\mathbf{B}$. For a gas pressure p and density ρ the component of the hydrodynamic equation in the direction of the flow becomes

$$v \frac{\partial v}{\partial l} + \frac{1}{\rho} \frac{\partial p}{\partial l} + \frac{\partial \Phi}{\partial l} = 0. \tag{1}$$

Assuming that the radius of curvature of the streamer is large compared to the diameter of the streamer, the force exerted on the gas by the field is only a transverse or lateral pressure. Hence, integration of the component of the hydrodynamic equation perpendicular to the flow leads to the condition

$$p + \frac{B^2}{8\pi} = \text{constant} \tag{2}$$

across the streamer at any fixed distance along the streamer. If $A(l)$ denotes the area of the cross section of the streamer, then conservation of matter yields

$$A(l)\, v(l)\, \rho(l) = A_o v_o \rho_o , \tag{3}$$

and conservation of magnetic flux yields

$$A(l)\, B(l) = A_o B_o \tag{4}$$

where the zero subscripts denote the value at some suitable reference point along the streamer.

Now the mathematical solution of (1) and (3) has been discussed elsewhere for general $A(l)$ (see appendix of Parker, 1963a) and at great length for the circumstance that the lines of flow are approximately radial with $A(l)$ proportional to r^2 (Parker, 1958b, 1960, 1963 a,b,c). We know from these earlier investigations that v becomes supersonic at large distances from the sun to produce the phenomenon of the solar wind. In the present case we consider the solution of (1) and (3) for approximately radial flow with $A(l)$ determined by the condition (3) for lateral pressure balance. Then $l \cong r$ and $\Phi = GM_{\odot}/r$. With the notation $w^2 \equiv GM_{\odot}/a$, $\xi = r/a$, and $n = N(r)/N_0$, $c^2 = 2kT(r)/M$, where M is the ion mass and $\rho = NM$, the momentum equation (1) may be written

$$v\frac{dv}{d\xi} + \frac{1}{n}\frac{d}{d\xi}nc^2 + \frac{w^2}{\xi^2} = 0 . \tag{5}$$

The present aim is to illustrate the differences that might be introduced in the expansion of the solar corona by possible filamentary structure. Thus we consider the extreme case that the gas density between the streamers is identically zero.*

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* Neglecting the gas density between the streamers implies that the density there at the base of the corona is extremely low and/or the mean temperature is somewhat lower.

Presumably then the actual situation lies somewhere between this extreme illustrative example and the earlier case $A(l) \propto r^2$ for a homogeneous corona. In the present extreme case, then, the condition (2) for lateral pressure equilibrium may be written

$$n(\xi)\,c^2(\xi) + \frac{B_1^2(\xi)}{8\pi N_0 M} = \frac{B_2^2(\xi)}{8\pi N_0 M} \tag{6}$$

where the subscripts 1 and 2 refer to the streamer and interstreamer regions respectively. The magnetic fields $B_1(\xi)$ and $B_2(\xi)$ are related to the fields B_{10} and B_{20} at the reference level $r = a$ by the continuity condition (4). So far as the cross-sectional areas $A_1(\xi)$ and $A_2(\xi)$ of the streamer and interstreamer are concerned, we note that taking an average around the sun, their sum must increase outward like r^2. Thus write

$$A_1(\xi) + A_2(\xi) \cong \Omega a^2 \xi^2 \tag{7}$$

as the mean condition around the sun. Here Ω is the mean solid angle per unit streamer and interstreamer region.*

II Streamer Geometry

The problem now is to work out the variation of the cross section of the streamer with radial distance from the sun and to see what effect this may have on the solar wind

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* The value of Ω is quite irrelevant and the reader may, if he wishes, consider a unit solid angle with $A_1(r)$ representing that portion of it occupied by streamers.

velocity and density in space. Use (4) to express $A(\xi)$ in terms of $B(\xi)$ and substitute into (7). Then solve (6) for $B_2(\xi)$ and eliminate $B_2(\xi)$ from the result of (7), obtaining the quartic equation

$$\left[B_{01} A_{01} - \Omega a^2 B_1(\xi)\xi^2\right]^2 \left[B_1^2(\xi) + 8\pi N_0 M n c^2\right] \tag{8}$$

$$- \left[B_{02} A_{02} B_1(\xi)\right]^2 = 0$$

whose solution gives $B_1(\xi)$ in terms of $n(\xi)\, c^2(\xi)$. The area $A_1(\xi)$ follows from (4), and $A_2(\xi)$ from (7). The form of $n(\xi)\, c^2(\xi)$, on which $A_1(\xi)$ and $A_2(\xi)$ depend directly, may be written down using the approximations developed in the first paper of this series (Parker, 1963b, hereafter referred to as paper I). From the base of the corona out to the critical point,

$$n(\xi)\, c^2(\xi) \cong c_0^2 \exp\left[- w^2 \int_1^{\xi} \frac{du}{u^2 c^2(u)}\right] . \tag{9}$$

Beyond the critical point use

$$n(\xi)\, c^2(\xi) = \frac{v_0\, c^2(\xi)}{v(\xi)\, \xi^2} \tag{10}$$

where $v(\xi)$ is a slowly varying function of position, approximated by eqn. (22) of paper I. The asymptotic relation $v(\xi) \sim v(\infty)$ is valid at large ξ.

It is sufficient for the present purposes to consider two special cases, $A_{01} >> A_{02}$ and $A_{01} << A_{02}$. These two extremes will illustrate the possibilities at hand. First of all, suppose that $A_{01} >> A_{02}$ Under these circumstances the interstreamer region has negligible cross section, and the streamer has a cross section

$$A_1(\xi) \cong \Omega a^2 \xi^2. \tag{11}$$

This, then, is the circumstance treated in earlier papers. There is no essential effect of small interstreamer regions on coronal expansion. The mean solar wind velocity and density are unchanged. The only difference is the presence of thin interstreamer regions with the small cross section

$$A_2(\xi) = A_{20} \frac{B_{20}}{\left[8\pi N_0 M n(\xi) c^2(\xi) + B_{10}^2 / \xi^4\right]^{1/2}}. \tag{12}$$

It is of some interest to examine (12) to determine whether the portion of space $A_2(\xi)$ occupied by the interstreamer regions increases or decreases in proportion to the streamer region $A_1(\xi)$ with increasing radial distance from the sun. Adiabatic cooling of the gas in the streamer probably represents the case for minimum streamer gas pressure, yielding the maximum interstreamer cross section. Thus, if the flow should be adiabatic beyond some distance ξ_1, the interstreamer cross section is

$$A_2(\xi) \sim A_{20}\left[\frac{B_{20}^2}{8\pi N_0 M\, n(\xi_1)\, c^2(\xi_1)}\right]^{1/2}\left(\frac{\xi}{\xi_1}\right)^{5/3} \tag{13}$$

for large ξ. We would expect, however, that the temperature may perhaps decline somewhat less rapidly than adiabatically as a consequence of thermal conduction, dissipation of disordered velocities, etc. (see discussion in Parker, 1963c, hereafter referred to as paper II) If it is assumed that the temperature declines as $1/\xi^{\alpha}$ ($\alpha < 1$, paper II) then for large ξ the interstreamer region increases outward like

$$A_2(\xi) \sim A_{20}\left[\frac{B_{20}^2}{8\pi N_0 M\, n(\xi_1)\, c^2(\xi_1)}\right]^{1/2}\left(\frac{\xi}{\xi_1}\right)^{1+\alpha/2}$$

In either case it is evident that $A_2(\xi)$ increases with ξ less rapidly than the streamer area $A_1(\xi)$. Thus if the interstreamer regions form a small fraction of the total near the sun, they constitute a small fraction at large distances too. The physical reason for this is the simple fact that the magnetic pressure declines more rapidly with expansion than does the gas pressure. Hence to maintain lateral pressure balance the magnetic interstreamer regions must not expand as rapidly as the gaseous streamers. The relative angular width of streamer and inter-streamer regions may be computed as a function of radial distance by employing (9) and (10).

Now consider the opposite extreme, $A_{01} << A_{02}$, wherein the streamers are small and the interstreamer region dominates. We have, then, that

$$A_2(\xi) \cong \Omega a^2 \xi^2 \tag{14}$$

and

$$A_1(\xi) = A_{10} \frac{B_{10}}{\left[B_{20}^2/\xi^4 - 8\pi N_0 M n(\xi) c^2(\xi)\right]^{1/2}} \tag{15}$$

The result is that the streamer $A_1(\xi)$ increases with ξ more rapidly than $A_2(\xi)$ and eventually dominates $\left(A_1(\xi) > A_2(\xi)\right)$ even though $A_1(\xi)$ is taken to be small near the sun. To see this, suppose that $A_1(\xi)$ increases only as fast as ξ^2. Then in the extreme case of adiabatic cooling $n(\xi)\, c^2(\xi)$ decreases like $1/\xi^{10/3}$. This decline is less rapid than the $1/\xi^4$ decline of the magnetic pressure. The denominator on the right hand side of (15) diminishes rapidly as a consequence of the increasing cancellation of the two terms, and $A_1(\xi)$ increases more rapidly than ξ^2 QED. A slower decline in $n(\xi)\, c^2(\xi)$ means a more rapid increase of A_1. The physical explanation for this is again the rapid decline of the magnetic pressure in the interstreamer region, requiring the streamers to broaden rapidly and diminish their density in order to maintain lateral pressure balance. It is evident that sufficiently far into space the extreme case of vanishing inter-streamer gas density leads to $A_1(\xi) \gg A_2(\xi)$ no matter how small may A_{10}/A_{20} be at the base of the corona.* The expansion velocity is unchanged in order of magnitude,

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* It must be remembered that the approximation on which (15) is computed is $A_1 \ll A_2$, so that (15) is not valid when A_1 becomes comparable to A_2.

of course, by the more rapid increase in $A_1(\xi)$. The principal effect is an enormous decrease in the density in the streamer region with the rapid increase of $A_1(\xi)$.

It is sufficient for the present purposes to restrict the numerical exposition to the idealized case of uniform temperature along the streamer. The angular width of the interstreamer region when $A_1 >> A_2$ is plotted in dimensionless form (see Appendix I) in Fig. 1. We note that the angular width is bounded and tends to decrease toward both large and small ξ. The angular width of the streamer when $A_1 << A_2$ is plotted in dimensionless form (see Appendix II) in Fig. 2. Note that the angular width increases without bound toward both large and small ξ, with a minimum between. As we will see in the next section, the minimum tends to choke off the mass flow, by a factor $(\nu^2 - 1)^{1/2}/\nu$ (ν is defined in (17)). The minimum is the result of the rapid decline of the hydrostatic pressure nc^2 with height, leading to a contraction of the streamer to enhance its magnetic pressure to make up the balance required by (6).

III. Expansion along a Streamer

With a knowledge of the streamer density and cross-section $A_1(\xi)$ it is now possible to compute the expansion velocity from the equation (3) for conservation of mass. There is no need to repeat the calculations for the case that the interstreamer regions occupy only a small fraction of the total solid angle $(A_{20} << A_{10})$ so that the streamer cross section increases as ξ^2. The results may be found in the literature (see for instance Parker, 1958, 1963, a, b, c). The opposite extreme $(A_{20} >> A_{10})$ is a new one, however, and will be worked out here to show the effect on the density and velocity of the wind in the streamers.

Again it will be sufficient for purposes of illustration to limit the discussion to uniform temperature along the streamer. Then using (3) to eliminate $n(\xi)$ from (5) gives

$$\frac{dv}{d\xi}\left(v - \frac{c_o^2}{v}\right) = \frac{c_o^2}{A_1}\frac{dA_1}{d\xi} - \frac{w^2}{\xi^2} .$$

The critical point occurs at the distance ξ_c given by the root of

$$\frac{\xi^2}{A_1}\frac{dA_1}{d\xi} - \frac{w^2}{c_o^2} = 0 . \tag{16}$$

The velocity has the usual value $v = c_o$ at the critical point. The approximation (9) is valid from the base of the corona $\xi = 1$ out to $\xi = \xi_c$. Let A_{10}/A_{20} be so small and ν^2 and w^2/c_o^2 be so large that A_1/A_2 remains small compared to one out at least as far as ξ_c (see discussion in appendix II). Then if we define the dimensionless parameter ν as

$$\nu^2 \equiv \frac{B_{20}^2}{8\pi N_o M c_o^2} , \tag{17}$$

so that

$$\frac{B_{10}}{B_{20}} = \frac{(\nu^2 - 1)^{1/2}}{\nu} ,$$

it is readily shown from (12) that (16) reduces to

$$\frac{2\nu^2}{\xi^3}\left(\frac{1}{\xi} - \frac{2c_0^2}{w^2}\right) = \exp\left[-\frac{w^2}{c_0^2}\left(1 - \frac{1}{\xi}\right)\right]. \tag{17}$$

Since $\nu^2 \geq 1$ and $w^2/c_0^2 >> 1$, there is a real positive root ξ_c to this equation and hence there is the usual critical point.

The important quantity to compute now is v_0 since this gives us the mass flow and the density of the solar wind in the streamer. To do this we must obtain an expression for ξ_c. Let

$$\xi_c \equiv \frac{w^2}{2c_0^2}\left[1 - \epsilon\left(\frac{w^2}{c_0^2}, \nu^2\right)\right], \tag{18}$$

where $\epsilon\left(w^2/c_0^2, \nu^2\right)$ is assumed to be small compared to one. Substituting into (17) yields

$$\epsilon\left(\frac{w^2}{c_0^2}, \nu^2\right) = \frac{1}{2\nu^2}\left(\frac{w^2}{2c_0^2}\right)^4 \exp\left(2 - \frac{w^2}{c_0^2}\right). \tag{19}$$

Note that $\epsilon \ll 1$, as assumed in (18), if w^2/c_o^2 is sufficiently large $(w^2/c_o^2 \gtrsim 10)$. This locates the critical point as $\xi_c \cong w^2/2c_o^2$, neglecting terms $O(\epsilon)$. The cross-section of the streamer at the critical point is readily shown from (15) to be

$$\frac{A_\perp(\xi_c)}{A_{\perp o}} = \left(\frac{\nu^2-1}{\nu^2}\right)^{1/2}\left(\frac{w^2}{2c_o^2}\right)^2\left[1+O(\epsilon)\right]. \tag{20}$$

The density at the critical point is computed from (9) to be

$$n(\xi_c) = \exp\left(2 - \frac{w^2}{c_o^2}\right). \tag{21}$$

Relating the velocities at $\xi = 1$ and $\xi = \xi_c$ with (3) it follows that

$$\frac{v_o}{c_o} \cong \left(\frac{\nu^2-1}{\nu^2}\right)^{1/2}\left(\frac{w^2}{2c_o^2}\right)^2 \exp\left(2 - \frac{w^2}{c_o^2}\right) \tag{22}$$

for the circumstance that the streamers occupy only a small portion of space. The same approximation (9) applied to the opposite extreme, where the interstreamer regions occupy a small portion of space, gives the same form for v_o/c_o except that the factor $(\nu^2-1)^{1/2}/\nu$ is missing. The density of the solar wind in a streamer at large radial distance from the sun is given by (3) in terms of v_0 as

$$n(\xi) = \frac{v_0 A_{10}}{v(\xi) A_1(\xi)} . \tag{23}$$

Thus the relative density $n(\xi)$ in space is lower when the streamers occupy only a small fraction of the volume near the sun because of the much greater increase of $A_1(\xi)/A_{10}$, discussed in the previous section, and because of the reduction of v_0 by the factor $(\nu^2 - 1)^{1/2}/\nu$. We will have more to say on this later.

IV. Streamer Instability

Consider the dynamical instabilities that may arise between coronal streamers. First of all there are the hose and mirror instabilities which occur when the thermal motions are sufficiently anisotropic (Parker, 1959a). The presence of anisotropy does not depend upon the existence of discrete streamers. For instabilities depending upon filamentary or streamerlike structure the most obvious is an exchange instability (Rosenbluth and Longmire, 1957) that may arise as a consequence of the magnetic field in the interstreamer region lying parallel, or nearly parallel, to the adjacent field in the streamer. An exchange of magnetic lines of force, and the associated plasma, involves very little expenditure of energy and approximates to a condition of neutral stability. The exchange may be expected to take place particularly if the magnetic pressure in the streamer $B_1^2/8\pi$ should be small compared to the gas pressure. In this case the streamer is effectively a tube of nearly field free gas confined by the parallel external interstreamer field. Consequently we would expect that if $B_1 << B_2$, the streamer would engulf some of the interstreamer field and make B_1 more nearly comparable to B_2. In any case, the exchange instability must lead to a softening of any extreme filamentary condition with increasing distance into space. So little is known about gas and field conditions in the corona that very little can be done to predict the effectiveness of the exchange instability on theoretical grounds. Observation will have to settle the matter.

Another class of instability that may be important is the Helmholtz instability, arising from the waves generated in the boundary between two streams with nonvanishing velocity difference. The idealized case of inviscid incompressible flow is considered in Appendix III. It is shown that two adjacent parallel streams of gas of comparable density are unstable in both the sausage and serpentine modes to disturbances with wavelengths of a few times the width of the streams, or shorter, whenever the relative velocity V of the streams somewhat exceeds the Alfvén speed computed in either stream. The growth time for the instability is then of the order of the time in which the velocity V carries the fluid a distance comparable to one wavelength. There is, of course, no instability between the streamer and interstreamer region for the idealized circumstance that the interstreamer gas density is identically zero.* But the actual streamers cannot be so sharp, and instability is expected.

* When the interstreamer density is identically zero, the only Helmholtz instability is between streamers with different streaming velocity. The disturbance is communicated between such streamers by the essentially massless interstreamer fields which lie between.

The Mariner II observations suggest that the density of the solar wind at the orbit of Earth is 1 - 5 protons/cm^3 (Neugebauer and Snyder, 1962) and the magnetic field is commonly 4×10^{-5} gauss (Smith, _et al_ 1963), yielding Alfven speeds of the general order of 50 km/sec. Extrapolating both the field density and the gas density back in toward the sun as $1/r^2$, it follows that the Alfven speed should increase like $1/r$ toward the sun, reaching, say, 150 km/sec at the orbit of Mercury. Now the solar wind velocity is observed to be of the order of 500 km/sec. We would expect that the solar wind velocity has this same value in at least as far as the orbit of Mercury. The velocity available for producing the instability is not the full solar wind velocity, of course, but rather the difference between the streamer and interstreamer wind velocities. We have no way of estimating this velocity difference, but if it should be as large as 100 km/sec, then we would perhaps expect instability, in the sausage and serpentine modes, somewhat before the orbit of Earth is reached.

We conclude that, if it should be the case that the solar corona is composed of streamers, then the streamer and interstreamer regions must become subject to Helmholtz instability in interplanetary space. Such instability may contribute to the breakup of the streamers, and to the irregularities* in the interplanetary magnetic field observed at Mariner II (Smith, _et al_, 1963). The scale of the irregularities will range from a few times the width of the streamer -- which unfortunately is not known -- down to scales so small as to be limited by diffusion, etc.

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* Further sources of irregularity have been discussed elsewhere (Parker, 1958a, 1963a)

V. Conclusions

The calculations presented in the previous sections illustrate the dynamical behavior to be expected of a corona that is more filamentary than homogeneous. It is not yet possible to state from observations to what extent the solar corona is confined to thin streamers and filaments. But we note that the extension into space of thin streamers in the corona may perhaps be the origin of the radial striations in the electron density observed by Hewish (1958; 1961) to a radial distance of about 100 $R_{\odot}$. Hewish concluded from his observations (of the polarization of the radio emission from the Crab Nebula as it passed by the sun) that the striations must be of large amplitude, $\Delta N \sim N$. This suggests, then, that if Hewish was in fact observing extended coronal streamers, those streamers had not broken up (by Helmholtz instability, etc.) as far out as 0.4 a.u. On the other hand, it is not evident that Mariner II observed any clear cut evidence of streamers between 0.7 and 1.0 a.u. Mariner II showed clear evidence of some disorder in the magnetic fields extending outward from the sun. The disorder may be the result of instabilities in ordered streamers closer to the sun. So we wonder if perhaps the instability becomes effective somewhere in the vicinity of 0.5 a.u., leaving observable striations closer to the sun and obliterating them beyond. This is at best a guess, of course, and the existence and behavior of streamers will probably have to be settled by observations in space to solar distances of 0.3 a.u. or less.

Consider the consequences of a filamentary coronal structure to the mean solar wind velocity and density observed at the orbit of Earth. The streamers occupy most of space at large radial distance from the sun no matter how slender and sparse they may be in the low corona. The expansion velocity for a given temperature distribution $T(r)$ outward from the sun is not greatly affected. If the streamers are exceedingly slender in the low corona the density ratio N / N_0 may be enormously reduced, compared to what it would be for a more homogeneous corona. However, coronal observations tend to give the mean density $\langle N_0 \rangle$ over both streamer and interstreamer regions, rather than the density N_0 in a streamer alone. For the extreme case considered in this paper, in which the interstreamer density is taken to vanish, the mean density is related to the streamer density by

$$\langle N_0 \rangle = N_0 \frac{A_{10}}{\Omega a^2} .$$

Then using (23), and remembering that $A_1(\xi) \sim \Omega a^2 \xi^2$ at large radial distance for all values of A_{10} / A_{20} in the low corona, we have

$$\frac{N}{\langle N_0 \rangle} = \frac{v_0}{v \xi^2} .$$

But this is the same algebraic relation that obtains for a homogeneous corona, in which $\langle N_0 \rangle = N_0$. The solar wind velocity v is about the same at large radial distance as for a homogeneous corona with the same temperature profile $T(r)$. The only difference is that v_0 is given by (22), which contains the factor $(\nu^2 - 1)^{1/2} / \nu$ that does not appear for a homogeneous corona. We expect that this factor, which is equal to B_{10} / B_{20}

is of the general order of unity because of the exchange instabilities that arise if it is very much less than one. Thus, we conclude that for a given mean coronal density $\langle N_0 \rangle$ the solar wind density N depends principally upon the temperature $T(r)$ -- to which it is rather sensitive -- and is, in order of magnitude, essentially independent of the degree of striation in the corona. The present calculations are for the extreme case of vacuum in the interstreamer regions. The actual case, where the interstreamer gas density is nonvanishing, must lie somewhere between the present result and the homogeneous corona; that is to say, for radial flow the actual case must have an $N / \langle N_0 \rangle$ for a given $T(r)$ which is close to the value for a homogeneous corona. Of course the filamentary structure may affect $T(r)$, and therefore indirectly affect N, but this is another problem, involving the theory of coronal heating and the heat flow equation. It is beyond the scope of the present paper. The direct effect of a filamentary coronal structure for a given $T(r)$ is a small change (less than a factor of two) in v_1, and a reduction of $N / \langle N_0 \rangle$ by a factor not exceeding the order of B_{10} / B_{20}.

In closing it should be noted that a number of effects were omitted in the discussion of the dynamics of a streamer corona, not the least of which are time variations and the consequences of solar rotation. It is to be hoped that these effects, along with many others, can be considered when direct observation of plasmas and fields in space can point the way. The problem of streamers from a rotating sun is of particular interest because it leads to an Archimedes spiral pattern in which the faster tubes of flow tend to extend outward more nearly along the radius and to overtake the slower tubes of flow. This has been discussed elsewhere (Parker, 1963a) and has been discussed recently by Sarabhai (1963) in connection with possible effects on the galactic cosmic ray particles.

Appendix I

The cross sectional area of the streamer is given by (11) for the case that $A_1(\xi) >> A_2(\xi)$. The area of the interstreamer region is given by (12), which may be written

$$Y(\varpi) = \frac{1}{\left(\frac{1}{\varpi^4} + \frac{K}{U\varpi^2}\right)^{1/2}}$$

for the isothermal case $c^2 = c_0^2$, upon using (3) to eliminate n and (6) to eliminate B_{10}. The symbol ϖ is the dimensionless radial distance

$$\varpi = \frac{c_0^2}{w^2}\xi ,$$

U is the dimensionless velocity

$$U^2 = \frac{1}{2}\frac{v^2}{c_0^2} ,$$

and $Y(\varpi)$ is the relative area

$$Y(\varpi) = \frac{A_2(\xi)}{A_{20}}\frac{c_0^4}{w^4}\left(\frac{\nu^2-1}{\nu^2}\right)^{1/2}$$

where

$$\nu^2 \equiv \frac{B_{20}^2}{8\pi N_0 M c_0^2}.$$

The parameter K is defined as

$$K = \frac{U_0 w^4}{c_0^4(\nu^2 - 1)}.$$

The velocity U is given by

$$U^2 - \ln U - 2\ln\varpi - \frac{1}{\varpi} = \frac{5}{2}\ln 2 - \frac{3}{2}$$

(see Parker, 1963a) and is plotted in Fig. 3 as a function of ϖ. The critical point lies at $U = 2^{-1/2}$, $\varpi = 1/2$. The relative angular radius $Y^{1/2}(\varpi)/\varpi$ of the streamer is plotted in Fig. 1 for various values of K and for the limiting case $\nu = 1$ $(B_{10} = 0)$. The smaller values of K correspond to smaller coronal temperatures.

Appendix II

The cross sectional area of the interstreamer region is given by (14) for the case that $A_1(\xi) \ll A_2(\xi)$. The area of the streamer is given by (9) and (15), and may be written

$$Z(\varpi) = \frac{1}{\left[\frac{S}{\varpi^4} - \exp\left(\frac{1}{\varpi}\right)\right]^{1/2}}$$

for the isothermal case $c^2 = c_0^2$, where $Z(\varpi)$ is the relative area

$$Z(\varpi) \equiv \frac{A_1(\xi)}{A_{10}} \frac{\exp(-w^2/2c_0^2)}{(\nu^2-1)^{1/2}},$$

and S is the dimensionless constant

$$S \equiv \nu^2 \left(\frac{c_0^2}{w^2}\right)^4 \exp\frac{w^2}{c_0^2}.$$

The angular width of the streamer $Z^{1/2}(\varpi)/\varpi$ is plotted in Fig. 2 for the cases $S = 30$ and 100, the larger value of S representing lower coronal temperatures and/or relatively stronger fields.

The location of the critical point, where $v = c_0$, is discussed in section III. It occurs at $\varpi \cong 1/2$. In order that A_1 remain small compared to A_2 (as assumed in the present approximation) as far out as the critical point, it is necessary that

$$A_{10} << \frac{\nu^2}{\nu^2 - 1} A_{20}$$

as is readily shown from (19) and (20).

The base of the corona lies at $\varpi = c_0^2/w^2 \lessapprox 0.1$, so that the cross sectional area of the streamer becomes a minimum somewhere above the base. The base lies at smaller ϖ in Fig. 2 the lower is the coronal temperature, with the result that the minimum cross section may become a serious constriction.

Appendix III

Consider the stability of a stream of density ρ_0 of incompressible, inviscid, infinitely conducting fluid flowing with velocity V in the x-direction and bounded by the planes $y = \pm a$. The stream contains the magnetic field $\mathbf{e}_x B_0$, where $\mathbf{e}_x$ is a unit vector in the x-direction. Let the space outside the stream, i.e. $y < -a$, $y > +a$ be filled with an incompressible, inviscid, conducting fluid of density ρ_s at rest, containing the magnetic field $\mathbf{e}_x B_s$. Suppose that a small perturbation is introduced, represented by the velocity $\mathbf{v}$ $(|\mathbf{v}| << U)$, the pressure p, and the magnetic field $\mathbf{b}$ $(|\mathbf{b}| << B_0, B_s)$

Put

$$\mathbf{v} = \nabla \times \mathbf{e}_z \, \phi(x, y, t),$$

$$\mathbf{b} = \nabla \times \mathbf{e}_z \, \varphi(x, y, t).$$

Denote quantities pertaining to the region $y < -a$ with a subscript one, quantities pertaining to $-a < y < +a$ with no subscript, and quantities pertaining to $y > +a$ with a subscript two. It is readily shown that the linearized equations of motion are

$$\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)\frac{\partial \phi}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x},$$

$$\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)\frac{\partial \phi}{\partial x} = +\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{B_0}{4\pi\rho_0}\nabla^2\varphi,$$

$$\frac{\partial}{\partial t}\frac{\partial \phi_{1,2}}{\partial y} = -\frac{1}{\rho_s}\frac{\partial p_{1,2}}{\partial x},$$

$$\frac{\partial}{\partial t}\frac{\partial \phi_{1,2}}{\partial x} = +\frac{1}{\rho_s}\frac{\partial p_{1,2}}{\partial y} + \frac{B_s}{4\pi\rho_s}\nabla^2 Q_{1,2} .$$

The equations for the time rate of change of the field perturbation are

$$\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right) Q = B_0 \frac{\partial \phi}{\partial x} ,$$

$$\frac{\partial Q_{1,2}}{\partial t} = B_s \frac{\partial \phi_{1,2}}{\partial x} .$$

To treat the boundary conditions, let $\eta_1(x, t)$ represent the displacement of the lower boundary of the stream from its equilibrium position $y = -a$, and $\eta_2(x, t)$ the displacement of the upper boundary from $y = +a$. It follows that for an observer just inside the stream boundary, $d\eta_{1,2}/dt$ must equal v_y, or

$$\frac{\partial \eta_{1,2}}{\partial t} + V\frac{\partial v_{1,2}}{\partial x} = -\frac{\partial \phi}{\partial x} .$$

For an observer just outside,

$$\frac{\partial \eta_{1,2}}{\partial t} = -\frac{\partial \phi_{1,2}}{\partial x} .$$

Finally, the condition for continuity of the pressure across $y = +a$ and $y = -a$ requires that

$$p_{1,2} + \frac{B_s}{4\pi}\frac{\partial Q_{1,2}}{\partial y} = p + \frac{B_0}{4\pi}\frac{\partial Q}{\partial y}$$

at the appropriate boundary. The requirement that the normal component of $\mathbf{B}$ be continuous across the boundaries is taken care of automatically by the conditions on $\eta_{1,2}$ and the fact that the magnetic lines of force move with the fluid.

Now put

$$\phi = \left[A_1 \exp(-ky) + A_2 \exp(+ky)\right] \exp i(kx - \omega t),$$

$$\phi_1 = A_3 \exp(+ky) \exp i(kx - \omega t),$$

$$\phi_2 = A_4 \exp(-ky) \exp i(kx - \omega t),$$

$$p = \left[P_1 \exp(-ky) + P_2 \exp(+ky)\right] \exp i(kx - \omega t),$$

$$p_1 = P_3 \exp(+ky) \exp i(kx - \omega t),$$

$$p_2 = P_4 \exp(-ky) \exp i(kx - \omega t),$$

$$Q = \left[C_1 \exp(-ky) + C_2 \exp(+ky)\right] \exp i(kx - \omega t),$$

$$Q_1 = C_3 \exp(+ky) \exp i(kx - \omega t),$$

$$Q_2 = C_4 \exp(-ky) \exp i(kx - \omega t),$$

$$\eta_{1,2} = D_{1,2} \exp i(kx - \omega t),$$

which satisfy the requirement that the perturbation vanishes for $y \longrightarrow \pm\infty$ if it is assumed that k is positive.

Note that $\nabla^2 Q = 0$, which implies that there are no volume forces exerted by the magnetic field. The magnetic forces are exerted only at the boundaries of the stream. Substituting the forms for ϕ and p into the equation of motion in the stream yields

$$P_1 = -\rho_0 (\omega - kV) A_1,$$

$$P_2 = +\rho_0 (\omega - kV) A_2.$$

Substituting the forms into the equations of motion of the fluid outside the stream yields

$$P_3 = +\omega\rho_s A_3, \quad P_4 = -\omega\rho_s A_4.$$

The equations for Q yield

$$C_{1,2} = -\frac{B_0 k}{\omega - kV} A_{1,2},$$

$$C_{3,4} = -\frac{B_s k}{\omega} A_{3,4}.$$

The boundary conditions at $y = \pm a$ yield

$$A_3 = \frac{\omega}{\omega - kV}\left(A_1 \exp 2ka + A_2\right),$$

$$A_4 = \frac{\omega}{\omega - kV}\left(A_1 + A_2 \exp 2ka\right)$$

after eliminating $D_{1,2}$. Finally, the boundary condition on the pressure yields

$$P_3 + C_3 \frac{kB_s}{4\pi} - P_1 \exp 2ka - P_2 + C_1 \frac{kB_0}{4\pi} \exp 2ka - C_2 \frac{kB_0}{4\pi} = 0,$$

$$P_4 - C_4 \frac{kB_s}{4\pi} - P_1 - P_2 \exp 2ka + C_1 \frac{kB_0}{4\pi} - C_2 \frac{kB_0}{4\pi} \exp 2ka = 0.$$

Suppose now that we direct our attention to the velocity perturbation in the stream. We express the other coefficients in terms of A_1 and A_2 and substitute into the two equations for the pressure boundary condition, obtaining finally

$$A_1 \left[\rho_s \omega^2 + \rho_0 (\omega - kV)^2 - \frac{k^2 (B_0^2 + B_s^2)}{4\pi}\right] \exp 2ka$$

$$+ A_2 \left[\rho_s \omega^2 - \rho_0 (\omega - kV)^2 + \frac{k^2 (B_0^2 - B_s^2)}{4\pi}\right] = 0,$$

$$A_1\left[\rho_s\omega^2-\rho_0(\omega-kV)^2+\frac{k^2(B_0^2-B_s^2)}{4\pi}\right]$$

$$+A_2\left[\rho_s\omega^2+\rho_0(\omega-kV)^2-\frac{k^2(B_0^2+B_s^2)}{4\pi}\right]\exp 2ka=0 .$$

Equating the determinant of the coefficients to zero leads to a quartic equation in ω , which breaks up into the two factors

$$\left(\rho_s\omega^2-\frac{k^2B_s^2}{4\pi}\right)\tanh ka+\left[\rho_0(\omega-kV)^2-\frac{k^2B_0^2}{4\pi}\right]=0$$

and

$$\left(\rho_s\omega^2-\frac{k^2B_s^2}{4\pi}\right)+\left[\rho_0(\omega-kV)^2-\frac{k^2B_0^2}{4\pi}\right]\tanh ka=0 .$$

Denote the roots of the first factor by ω_+ and the second by ω_- . Then

$$\omega_+=\frac{kV}{1+\alpha\tanh ka}\left\{1\pm\left[(1+\alpha\tanh ka)\left(\frac{C_0^2+\alpha C_s^2\tanh ka}{V^2}\right)\right.\right.$$

$$\left.\left.-\alpha\tanh ka\right]^{1/2}\right\} ,$$

$$\omega_- = \frac{kV}{1+\alpha \coth ka}\left\{1 \pm \left[\left(1+\alpha\coth ka\right)\left(\frac{C_0^2+\alpha C_s^2 \coth ka}{V^2}\right)\right.\right.$$

$$\left.\left. - \alpha \coth ka\right]^{1/2}\right\}$$

where $\alpha = \rho_s/\rho_0$ and C_0 and C_s are the Alfven velocities

$$C_0^2 = \frac{B_0^2}{4\pi\rho_0}\ , \qquad C_s^2 = \frac{B_s^2}{4\pi\rho_s}\ .$$

It is evident that when $\omega = \omega_+$ we have $A_1 = -A_2$, so that ϕ is an odd function of y. It follows that v_x is an even function of y, and v_y is an odd function of y, so that the mode corresponds to a variation in the width of the stream while the center line of the stream remains straight, resembling a string of sausages. The other mode $\omega = \omega_-$ yields $A_1 = A_2$ so that ϕ is an even function of y, and v_x is then odd and v_y even. This leads to a serpentine mode in which the width of the stream is preserved while the stream follows a sinusoidal path.

The sausage mode is unstable provided that

$$\alpha V^2 \tanh ka > (1+\alpha\tanh ka)(C_0^2 + \alpha C_s^2 \tanh ka)\,.$$

The serpentine mode is unstable provided that

$$\alpha V^2 \coth ka > (1 + \alpha \coth ka)(C_o^2 + \alpha C_s^2 \coth ka) .$$

It is evident for finite α and ka that in both cases the stream becomes unstable if the stream velocity V becomes sufficiently large compared to the Alfven speeds C_o and C_s. In the limit as α becomes small* the requirements for instability become

$$V^2 > \frac{1}{\alpha} (C_o^2 \coth ka + \alpha C_s^2)$$

and

$$V^2 > \frac{1}{\alpha} (C_o^2 \tanh ka + \alpha C_s^2)$$

for the sausage and serpentine modes respectively. The serpentine mode is the more unstable. In the limit as α becomes large instability requires that

$$V^2 > C_o^2 + \alpha C_s^2 \tanh ka$$

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* Note that as $\alpha \rightarrow 0$ we have $\alpha C_s^2 = B_s^2/4\pi\rho$ and is of the same order as C_o^2.

and

$$V^2 > C_0^2 + \alpha C_s^2 \coth ka ,$$

The sausage mode is the more unstable in this case. It is evident that the condition on V for instability becomes more severe as α becomes either very large or very small. The idealized streamers treated in the text, with vanishing density $\rho_s = 0$ in the interstreamer region, are not unstable for any value of V. In the limit of long wave lengths, $ka \longrightarrow 0$, the requirement is

$$V^2 > \frac{C_0^2}{\alpha k a}$$

in both cases. In the limit of short wave length, $ka \longrightarrow \infty$, the problem reduces to the disturbance in the plane boundary between two semi-infinite bodies of fluid with relative motion V, and the instability condition becomes

$$V^2 > \left(\frac{1+\alpha}{\alpha}\right)\left(C_0^2 + \alpha C_s^2\right) .$$

It is evident from the foregoing formulae and discussion that instability is most likely when ρ_s / ρ_o is of the order of unity and when the wavelength of the disturbance is not long compared to the width of the stream. When V is a factor of ten larger than the Alfven velocity, as it is in the observed solar wind, then all wavelengths comparable to, or shorter than, the width of the streamer are unstable, in both the sausage and serpentine modes. The characteristic growth time is of the order of the time in which the stream flows one wavelength.

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FIGURE CAPTIONS

Figure 1 A plot of the relative angular width $Y^{1/2}/\varpi$ of the interstreamer region as a function of the dimensionless distance ϖ from the sun for the case that the coronal temperature is uniform and the interstreamer regions are thin compared to the streamers. The parameter K represents the conditions of coronal temperature and the ratio of the streamer to the interstreamer magnetic fields. The curve labelled $\nu = 1$ is a plot of the relative angular width $(A_2/A_{20})^{1/2}(8\pi N_0 M c_0^2/B_{20}^2)^{1/4}/\varpi = U^{1/4}/\varpi^{1/2}$ for the limiting case that $B_{10} \rightarrow 0$.

Figure 2 A plot of the relative angular width $Z^{1/2}/\varpi$ of the streamer as a function of the dimensionless distance ϖ from the sun for the case that the coronal temperature is uniform and the streamer regions are thin compared to the interstreamer regions. The parameter S represents the conditions of coronal temperature and the ratio of the streamer to the interstreamer magnetic fields.

Figure 3 A plot of the dimensionless coronal expansion velocity $U(\varpi)$ for uniform coronal temperature. Note the different scales of radial distance ϖ for two curves. The lower curve and scale represent $U(\varpi)$ at small radial distance.

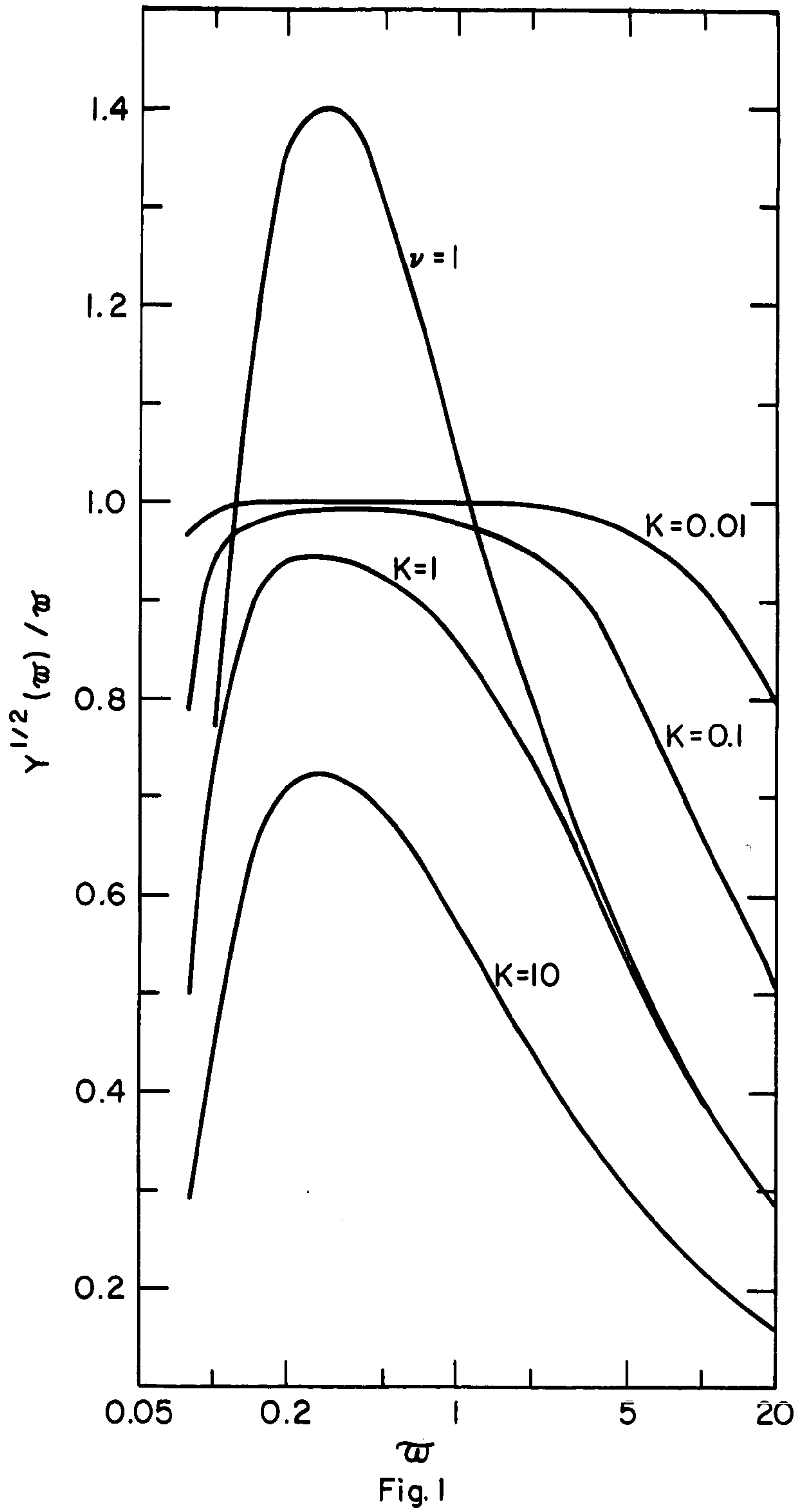
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$Y^{1/2}(\varpi)/\varpi$
$\nu=1$
K=0.01
K=1
K=0.1
K=10
0.05
0.2
1
5
20
ϖ

Fig. 1

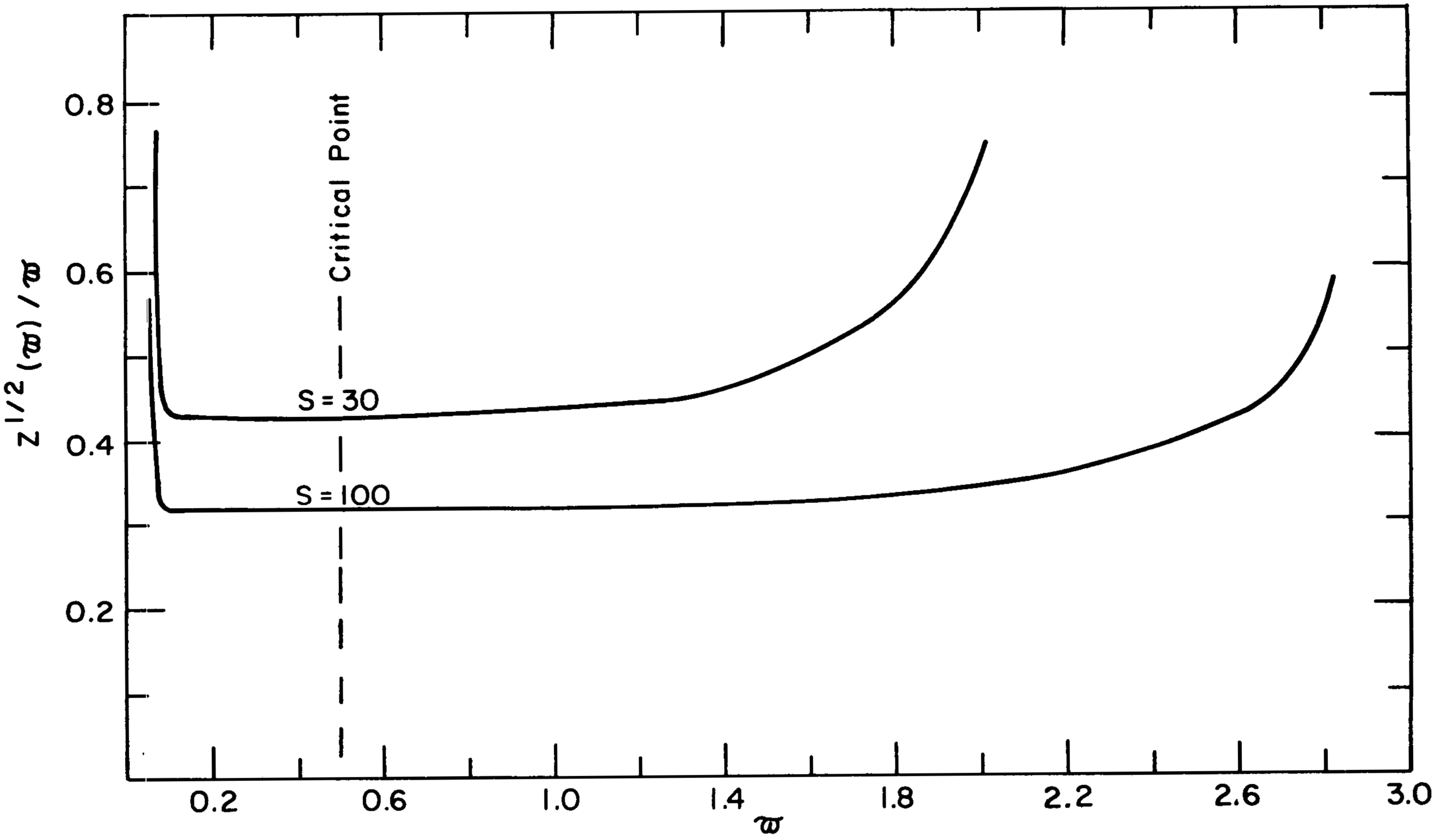
0.8
0.6
0.4
0.2
$Z^{1/2}(\varpi)/\varpi$
Critical Point
S = 30
S = 100
0.2
0.6
1.0
1.4
1.8
2.2
2.6
3.0
ϖ

Fig. 2

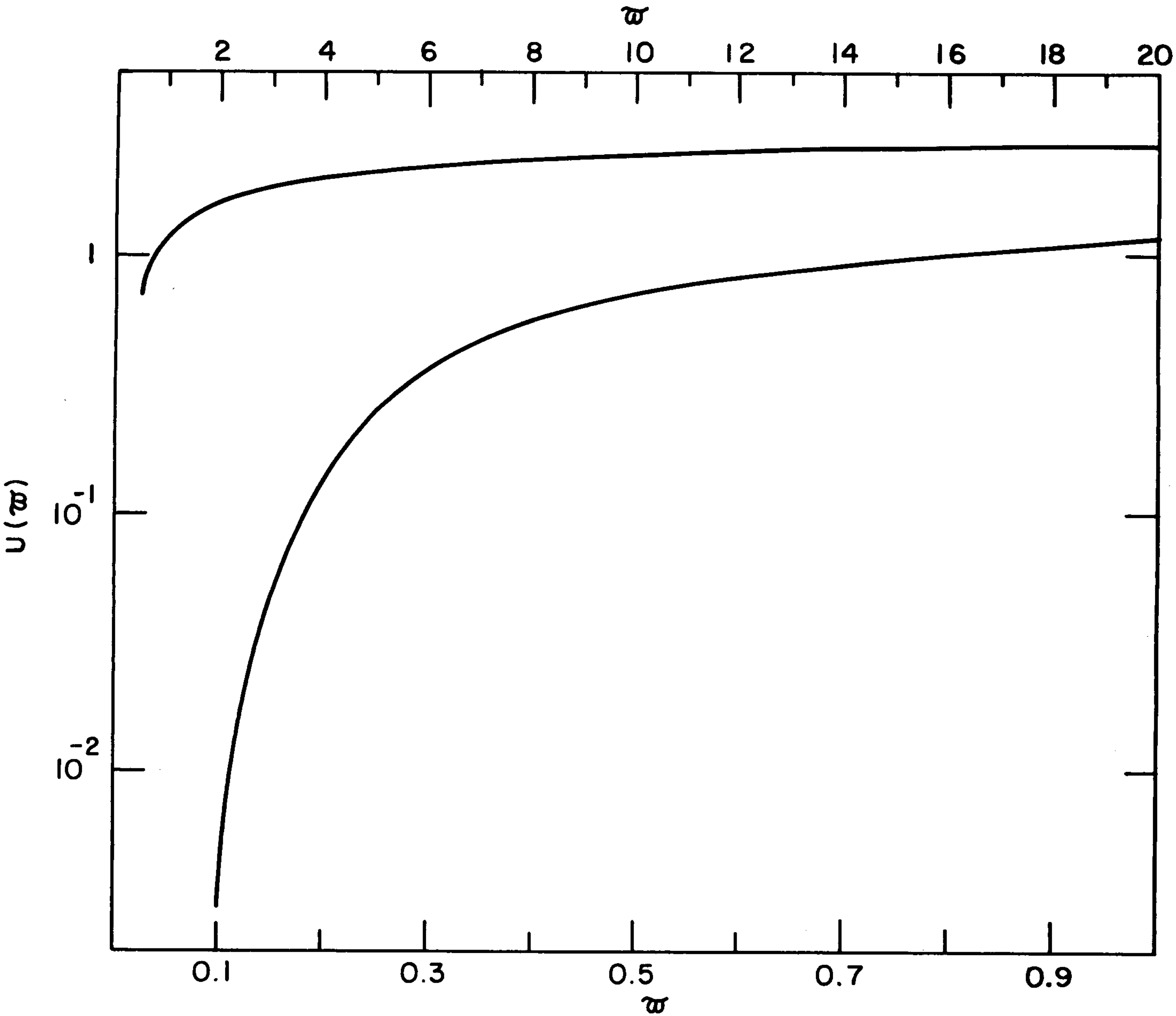

Fig. 3